SUPER GEOMETRIC MEAN LABELING OF SOME UNION OF GRAPHS

V. Hemalatha, V. Mohanaselvi

ABSTRACT-Let G be a graph with p vertices and q edges. Let $f:V(G)\to\{1,2,3,...,p+q\}$ be a injective function. For a vertex labeling f, the induced edge labeling $f(e=uv)$ is defined by $f(e)=\left\lceil\sqrt{f(u)f(v)}\right\rceil (or) \left\lfloor\sqrt{f(u)f(v)}\right\rfloor$. Then f is called a Super Geometric mean labeling if $f\left(V(G)\right)\cup\left\{f(e)/e\in E(G)\right\}=\{1,2,3,....,p+q\}$. A graph which admits Super Geometric mean labeling is called Super Geometric mean graph. In this paper, we investigate Super geometric mean labeling of some union of graphs.

Keywords: Graph, Super Geometric mean labeling, Super Geometric mean graph, union and arbitrary union.

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1 INTRODUCTION

We begin with simple, finite, connected and undirected graph G (V, E) with p vertices and q edges. For a detailed survey of graph labeling we refer to Gallian [1]. Terms are not defined here are used in the sense of Harary [2]. S. Somasundram and R. Ponraj introduced mean labeling of graphs in [6][7]. R.Ponraj and D. Ramya introduced Super mean labeling of graphs in [5]. S. Somasundram, P. Vidhyarani and R. Ponraj introduced Geometric mean labeling of graphs in [8].

Let $f:V(G)\to\{1,2,3,...,p+q\}$ be a injective function. For a vertex labeling f, the induced edge labeling $f(e=uv)$ is defined by $f(e)=\left\lceil\sqrt{f(u)f(v)}\right\rceil (or) \left\lfloor\sqrt{f(u)f(v)}\right\rfloor$. Then f is called a Super Geometric mean labeling if $f\left(V(G)\right)\cup\left\{f(e)/e\in E(G)\right\}=\{1,2,3,....,p+q\}$.

A graph which admits Super Geometric mean labeling is called Super Geometric mean graph.

2 PRIOR RESULTS

Theorem 2.1 [4] The path P_n is Super Geometric mean graph for any $n \geq 2$.

Theorem 2.2 [4] The Cycle C_n is a Super Geometric mean graph for any $n \geq 3$.

Theorem 2.3 [4] The Comb $P_n \odot K_1$, $(n \geq 2)$ is a Super Geometric mean graph.

Theorem 2.4 [4] The Dragon's C_m @ P_n are a Super Geometric mean graph.

Theorem 2.5 [4] The Triangular snake T_n are a Super Geometric mean graph.

Theorem 2.6 [3] The Flag Fl_m graph is a super geometric mean graph.

Theorem 2.7 [3] The graph $D_{n,m}$ is a Super Geometric mean graph for any $n,m\geq 3$.

Theorem 2.8 [3] The Kayak Paddle $KP(n,m,t)$ is a Super Geometric mean graph for $n,\ m\ \geq\ 3 and\ t\ \geq 1$.

Theorem 2.9 [3] The graph Polygonal snake $G_{m,n}$ is a Super Geometric mean graph.

Theorem 2.10 [3] The graph $\langle C_n:m\rangle$ where $n\geq 3\ and\ m\geq 1$ is a Super Geometric mean graph.

3 MAIN RESULTS

Definition 3.1 $G_1\cup G_2$ is nothing but disjoint union of two graphs G_1 and G_2.

Theorem 3.2 The graph $P_m\cup P_n$ is a Super Geometric Mean graph.

Proof

Let the vertices of $P_m\cup P_n$ be $\{u_i:1\leq i\leq m\}$ & $\{v_i:1\leq i\leq n\}$ the edges of $P_m\cup P_n$ be $\{e_i=u_iu_{i+1}:1\leq i\leq m-1\}$ & $\{e_i'=v_iv_{i+1}:1\leq i\leq n-1\}$.

Define a function $f\ :\ V\left(P_m\cup P_n\right)\to\ \{1,2,\ \ldots\ ,\ p+q\}$ by

$$f(u_i)=2i-1, \qquad 1\leq i\leq m$$
$$f(v_i)=2(m+i-1), \qquad 1\leq i\leq n$$

Then the induced edge labels are

$$f(e_i)=2i, \qquad 1\leq i\leq m-1$$
$$f(e_i')=2(m+i)-1, \qquad 1\leq i\leq n-1$$

Thus both vertices and edges together get distinct labels from $\{1,2,\ \ldots\ ,\ p+q\}$.

Hence $P_m\cup P_n$ is a Super Geometric mean graph.

Theorem 3.3 The graph $C_m\cup C_n$ is a Super Geometric Mean

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graph.

Proof

Let the vertices of $C_m \cup C_n$ be $\{u_i : 1 \le i \le m\}$ & $\{v_i : 1 \le i \le n\}$ the edges of $C_m \cup C_n$ be $\{e_i = u_i u_{i+1} : 1 \le i \le m\}$ & $\{e_i^{'} = v_i v_{i+1} : 1 \le i \le n\}$.

Define a function $f : V(P_m \cup P_n) \to \{1, 2, \dots, p+q\}$ by

$$f(u_i) = \begin{cases} 1 & i = 1 \\ 4i-2 & 2 \le i \le \frac{m+1}{2},\ if\ m\ is\ odd \\ & \&\ 2 \le i \le \frac{m}{2},\ if\ m\ is\ even \\ 4(m-i+1) & \frac{m+3}{2} \le i \le m,\ if\ m\ is\ odd \\ & \&\ \frac{m+2}{2} \le i \le m,\ if\ m\ is\ even \end{cases}$$

$$f(v_i) = \begin{cases} 2m+1 & i = 1 \\ 2(m+2i-1) & 2 \le i \le \frac{n+1}{2},\ if\ n\ is\ odd \\ & \&\ 2 \le i \le \frac{n}{2},\ if\ n\ is\ even \\ 2m+4(n-i+1) & \frac{n+3}{2} \le i \le n,\ if\ n\ is\ odd \\ & \&\ \frac{n+2}{2} \le i \le n,\ if\ n\ is\ even \end{cases}$$

Then the induced edge labels are

$$f(e_i) = \begin{cases} 4i-1 & 1 \le i \le \frac{m}{2},\ if\ m\ is\ even \\ & \&1 \le i \le \frac{m-1}{2},\ if\ m\ is\ odd \\ 4m-4i+1 & \frac{m+2}{2} \le i \le m-1,\ if\ m\ is\ even \\ & \frac{m+1}{2} \le i \le m-1,\ if\ m\ is\ odd \\ 2 & i = m \end{cases}$$

$$f(e_i^{'}) = \begin{cases} 2m+4i-1 & 1 \le i \le \frac{n}{2},\ if\ n\ is\ even \\ & \&1 \le i \le \frac{n-1}{2},\ if\ n\ is\ odd \\ 2m+4n-4i+1 & \frac{n+2}{2} \le i \le n-1,\ if\ n\ is\ even \\ & \&\frac{n+1}{2} \le i \le n-1,\ if\ n\ is\ odd \\ 2(m+1) & i = n \end{cases}$$

Thus both vertices and edges together get distinct labels from $\{1, 2, \dots, p+q\}$.

Hence $C_m \cup C_n$ is a Super Geometric mean graph.

Theorem 3.4 The graph $C_m \cup P_n$ is a Super Geometric Mean graph.

Proof

Let the vertices of $C_m \cup P_n$ be $\{u_i : 1 \le i \le m\}$ & $\{v_i : 1 \le i \le n\}$ the edges of $C_m \cup P_n$ be $\{e_i = u_i u_{i+1} : 1 \le i \le m\}$ & $\{e_i^{'} = v_i v_{i+1} : 1 \le i \le n-1\}$.

Define a function $f : V(C_m \cup P_n) \to \{1, 2, \dots, p+q\}$ by

$$f(v_i) = 2m+2i-1 \qquad 1 \le i \le n$$

$$f(u_i) = \begin{cases} 1 & i = 1 \\ 4i-2 & 2 \le i \le \frac{m+1}{2},\ if\ m\ is\ odd \\ & \&\ 2 \le i \le \frac{m}{2},\ if\ m\ is\ even \\ 4(m-i+1) & \frac{m+3}{2} \le i \le m,\ if\ m\ is\ odd \\ & \&\frac{m+2}{2} \le i \le m,\ if\ m\ is\ even \end{cases}$$

Then the induced edge labels are

$$f(e_i) = \begin{cases} 4i-1 & 1 \le i \le \frac{m}{2},\ if\ m\ is\ even \\ & \&1 \le i \le \frac{m-1}{2},\ if\ m\ is\ odd \\ 4m-4i+1 & \frac{m+2}{2} \le i \le m-1,\ if\ m\ is\ even \\ & \frac{m+1}{2} \le i \le m-1,\ if\ m\ is\ odd \\ 2 & i = m \end{cases}$$

$$f(e_i^{'}) = 2(m+i) \qquad 1 \le i \le n-1$$

Thus both vertices and edges together get distinct labels from $\{1, 2, \dots, p+q\}$.

Hence $C_m \cup P_n$ is a Super Geometric mean graph.

Theorem 3.5 The arbitrary union of paths is Super Geometric Mean graph.

Proof

Let the vertices of given graph be $\{v_{ij} : 1 \le i \le m\ \&\ 1 \le i \le n\}$ the edges be $\{e_{ij} : 1 \le i \le m\ \&\ 1 \le i \le n-1\}$.

Define a function $f : V(G) \to \{1, 2, \dots, p+q\}$ by

$$f(v_{ij}) = 2(n-1)(i-1) + 2j - 1 \qquad 1 \le i \le m\ \ \&\ 1 \le i \le n$$

Then the induced edge labels are

$$f(e_{ij}) = 2(n-1)(i-1) + 2j, \qquad 1 \le i \le m\ \ \&\ 1 \le i \le n-1$$

Thus both vertices and edges together get distinct labels from $\{1, 2, \dots, p+q\}$.

Hence the given graph is a Super Geometric mean graph.

Theorem 3.6 The arbitrary union of cycles is Super Geometric Mean graph.

Proof

Let $\{v_{ij} : 1 \le i \le m\ and\ 1 \le j \le n\}$ be the vertices and $\{e_{ij} : 1 \le i \le m\ and\ 1 \le j \le n\}$ be the edges of the given graph.

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Define a function $f : V(G) \rightarrow \{1, 2, \ldots, p+q\}$ by

$$f(v_{ij}) = \begin{cases} 2n(i-1)+1 & 1 \le i \le m,\ j=1 \\ 2n(i-1)+4j-2 & 1 \le i \le m,\ 2 \le j \le \frac{n+1}{2},\ \text{if } n \text{ is odd} \\ & \& \ 2 \le j \le \frac{n}{2},\ \text{if } n \text{ is even} \\ 2n(i-1)+4(n-j+1) & 1 \le i \le m,\ \frac{n+3}{2} \le j \le n,\ \text{if } n \text{ is odd} \\ & \& \ \frac{n+2}{2} \le j \le n,\ \text{if } n \text{ is even} \end{cases}$$

Then the induced edge labels are

$$f(e_{ij}) = \begin{cases} 2n(i-1)+4j-1 & 1 \le i \le m,\ 1 \le j \le \frac{n}{2},\ \text{if } n \text{ is even} \\ & \& 1 \le j \le \frac{n-1}{2},\ \text{if } n \text{ is odd} \\ 2n(i-1)+4n-4j+1 & 1 \le i \le m,\ \frac{n+2}{2} \le j \le n-1,\ \text{if } n \text{ is even} \\ & \& \frac{n+1}{2} \le j \le n-1,\ \text{if } n \text{ is odd} \\ 2n(i-1)+2 & 1 \le i \le m,\ j=n \end{cases}$$

Thus both vertices and edges together get distinct labels from $\{1, 2, \ldots, p+q\}$.

Hence the arbitrary union of cycles is a Super Geometric mean graph.

CONCLUSION

- If G_1 and G_2 are two Super Geometric Mean Graph, then union of two graphs are also Super Geometric Mean Graph.
- If G is a Super Geometric Mean Graph, then G-e is a Super Geometric Mean Graph.

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